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**LUMPED MASS FORMULATIONS
FOR
MODELING FLEXIBLE BODY SYSTEMS**

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ABSTRACT

This paper presents the efforts of Mechanical Dynamics, Inc. in obtaining a general formulation for flexible bodies in a multibody setting. The efforts being supported by MDI, both in house and externally are summarized. The feasibility of using lumped mass approaches to modeling flexibility in a multibody dynamics context is examined. The kinematics and kinetics for a simple system consisting of two rigid bodies connected together by an elastic beam are developed in detail. Accuracy, efficiency and ease of use using this approach are some of the issues that are then looked at.

The formulation is then generalized to a "superelement" containing several nodes and connecting several bodies. Superelement kinematics and kinetics equations are developed.

The feasibility and effectiveness of the method is illustrated by the use of some examples illustrating phenomena common in the context of spacecraft motions.

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SCOPE OF THE PRESENTATION

- **PROFILE OF MECHANICAL DYNAMICS, INC.**
- **MDI EFFORTS TO MODEL FLEXIBILITY**
- **LUMPED MASS APPROACHES TO FLEXIBILITY**
- **EXAMPLES**

PROFILE OF MECHANICAL DYNAMICS, INC.

- **COMPANY BACKGROUND**
 - **HISTORY**
 - **PRODUCTS & SERVICES**
 - **CUSTOMERS**
- **CURRENT PRODUCTS**
 - **ADAMS**
 - **ADAMS / MODAL**
 - **POST PROCESSORS**
- **SERVICES**
 - **CONSULTING**
 - **TRAINING**
 - **HOTLINE**
- **AVAILABILITY OF PRODUCTS**

MDI EFFORTS IN FLEXIBILITY

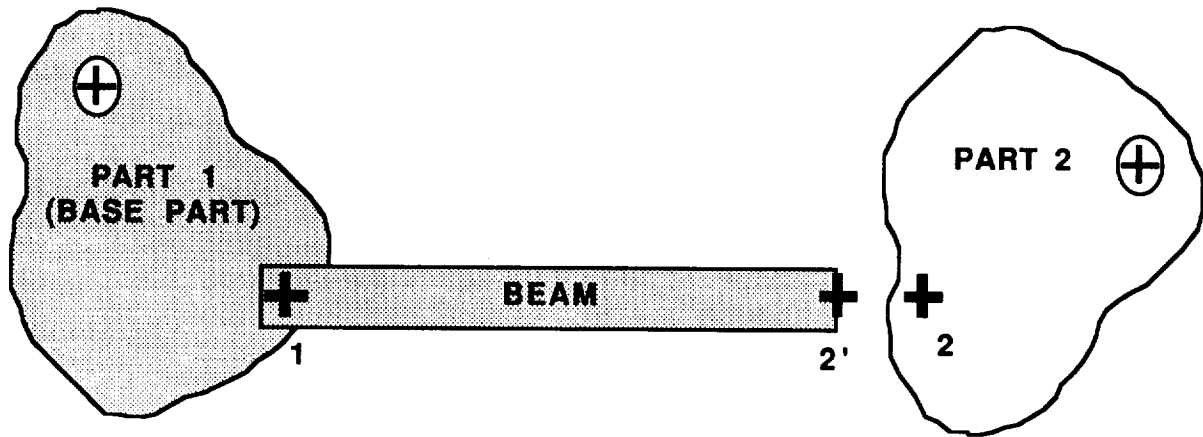
- **INTERNAL R&D**
 - **LUMPED MASS APPROACHES TO FLEXIBILITY**

- **EXTERNAL R&D**
 - **UNIVERSITY FUNDED RESEARCH IN MODAL APPROACHES**

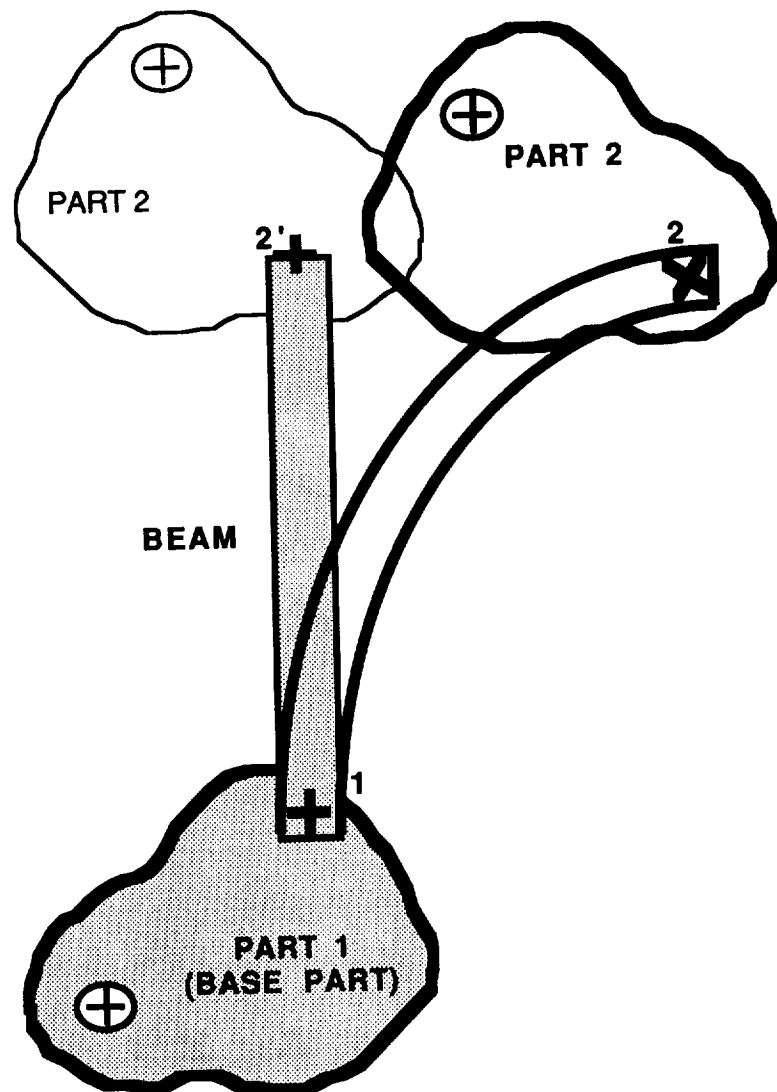
- **INTERFACE TO FEA PROGRAMS**
 - **NASTRAN**
 - **ANSYS**

- MECHANICAL DYNAMICS, INC. -

LUMPED MASS APPROACH TO FLEXIBILITY

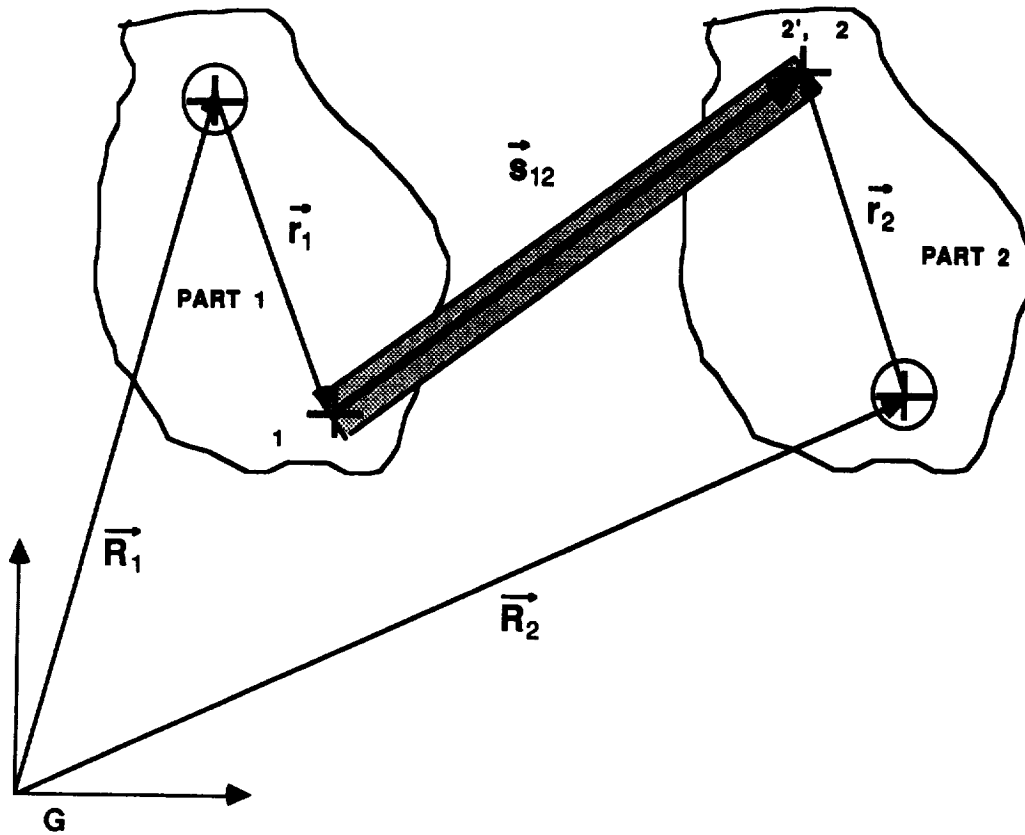


EXPLODED VIEW OF INITIAL CONFIGURATION



BEAM DEFORMATION DURING MOTION

TRANSLATIONAL DISPLACEMENT COMPUTATION



$$\vec{S}_{12} = \vec{R}_2 + \vec{r}_2 - \vec{R}_1 - \vec{r}_1$$

$${}^1\mathbf{S}_{12} = \mathbf{A}^{1G} [\mathbf{R}_2 + \mathbf{A}^{2G}\mathbf{r}_2 - \mathbf{R}_1 - \mathbf{A}^{1G}\mathbf{r}_1]$$

$${}^1\Delta^{2'2} = {}^1\mathbf{S}_{12} \cdot {}^1\mathbf{L}$$

$${}^1\mathbf{L} = \{ L \ 0 \ 0 \}^T$$

ANGULAR DISPLACEMENT COMPUTATION

SPACE 1-2-3 ANGLES ARE USED FOR MEASURING ANGLES

$$\mathbf{A}^{2'2} = \begin{bmatrix} C_2 C_3 & S_1 S_2 C_3 - S_3 C_1 & C_1 S_2 C_3 + S_3 S_1 \\ C_2 S_3 & S_1 S_2 S_3 - C_3 S_1 & C_1 S_2 S_3 + C_3 S_1 \\ -S_2 & S_1 C_2 & C_1 C_2 \end{bmatrix}$$

$$\beta_2 = \text{SIN}^{-1}(-a_{31})$$

IF $\beta_2 \neq \pi/2$ THEN

$$\beta_1 = \text{ARCTAN2}(a_{32}, a_{33})$$

$$\beta_3 = \text{ARCTAN2}(a_{12} + a_{13}, a_{13} \cdot a_{22}) - \beta_1$$

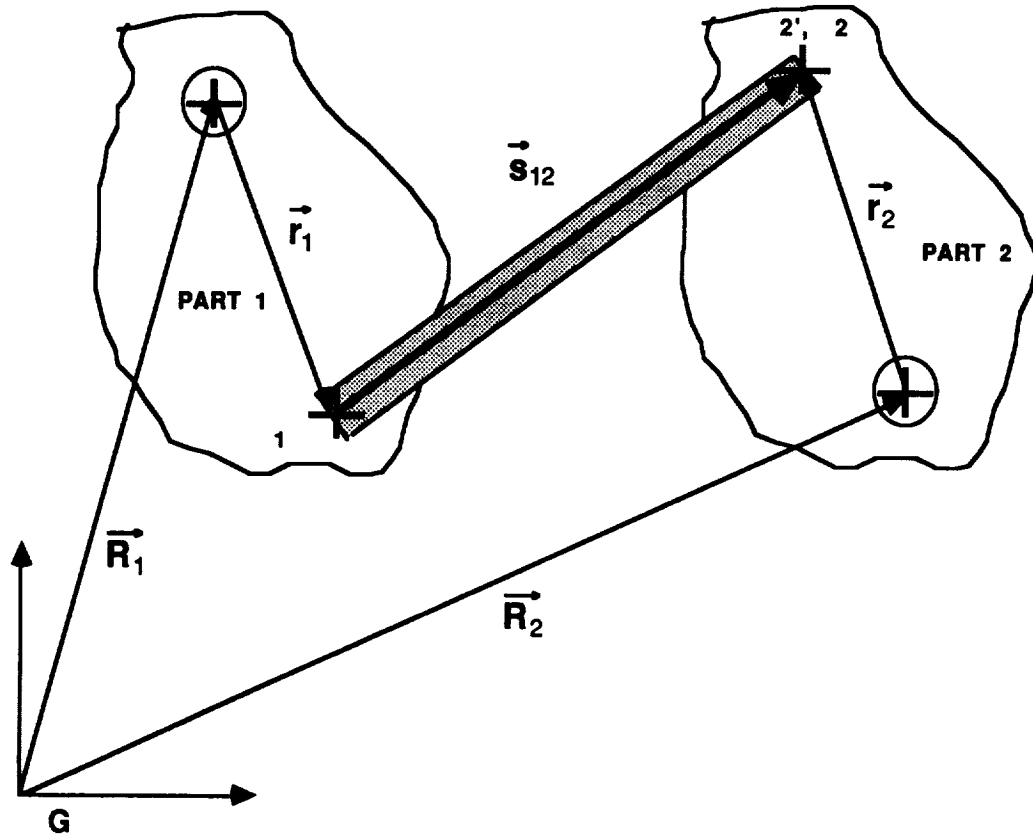
ELSE IF $\beta_2 = \pi/2$ THEN

$$\beta_3 = \text{ARCTAN2}(a_{21}, a_{11})$$

$$\beta_3 = \text{ARCTAN2}(a_{12}, a_{13}) + \beta_3$$

$$\mathbf{B}^{2'2} = [\beta_1 \ \beta_2 \ \beta_3]^T$$

VELOCITY COMPUTATION



$${}^{(1)}\vec{V}_{22} = (G)\vec{V}_{21} - \vec{\omega}_1 \times \vec{S}_{21}$$

$${}^1\underline{V}_{2'2} = \underline{A}^{IJ} [\underline{A}^{IG} (\underline{R}_I - \underline{R}_J) - \underline{A}^{JI} \underline{\omega}_I \underline{r}_1 - \underline{\omega}_J (\underline{A}^{JI} \underline{r}_1 + \underline{A}^{JG} \underline{R}_1 - \underline{A}^{JG} \underline{R}_1)]$$

$$\vec{\omega}_{12} = \vec{\omega}_2 - \vec{\omega}_1$$

$${}^1\omega_{12} = \underline{A}^{IJ} [\underline{A}^{JI} \omega_I - \omega_J]$$

FORCE COMPUTATION

FORCES AT ORIGIN OF COORDINATE SYSTEM ON REF. FRAME 1

$${}^1\mathbf{F}_2 = \begin{aligned} & - [\mathbf{K}_{11} \quad {}^1\Delta_{2'2} + \mathbf{K}_{12} \quad \mathbf{R}_{12}] \\ & - [\mathbf{C}_{11} \quad {}^1\mathbf{y}_{2'2} + \mathbf{C}_{12} \quad {}^1\omega_{12}] \end{aligned}$$

$${}^1\mathbf{T}_2 = \begin{aligned} & - [\mathbf{K}_{21} \quad {}^1\Delta_{2'2} + \mathbf{K}_{22} \quad \mathbf{R}_{12}] \\ & - [\mathbf{C}_{21} \quad {}^1\mathbf{y}_{2'2} + \mathbf{C}_{22} \quad {}^1\omega_{12}] \end{aligned}$$

K IS THE STANDARD MATRIX FOUND IN ANY STRUCTURAL ANALYSIS TEXT

FORCES AT ORIGIN OF COORDINATE SYSTEM ON REF. FRAME 2

SINCE THE BEAM IS MASSLESS, APPLYING LAWS OF EQUILIBRIUM :

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \vec{\mathbf{0}}$$

$${}^1\mathbf{F}_1 = - {}^1\mathbf{F}_2$$

$$\vec{\mathbf{T}}_1 + \vec{\mathbf{T}}_2 + \vec{\mathbf{S}}_{12} \times \vec{\mathbf{F}}_2 = \vec{\mathbf{0}}$$

$${}^1\mathbf{T}_1 = - [{}^1\mathbf{T}_2 + {}^1\tilde{\mathbf{s}}_{12} \quad {}^1\mathbf{F}_2]$$

ACCURACY OF METHOD

- **DIRECTLY RELATED TO DEGREE OF DISCRETIZATION**
- **METHOD DOES NOT YIELD WRONG ANSWERS**
- **DEGREE OF DISCRETIZATION DEPENDENT ON FREQUENCY**
CONTENT DESIRED. ADAMS/MODAL WILL COMPUTE
EIGENVALUES AND EIGENVECTORS FOR ANY ADAMS MODEL.
CAN ANIMATE LINEAR MODEL USING SELECTED SET OF MODE
SHAPES AND FREQUENCIES.

FLEXIBLE BODY DYNAMICS *Benchmark Problem #1: Uniform Cantilever Beam Spin-up*

The inset of Fig. 1 shows a uniform, homogeneous, cantilever beam supported by a circular hub of radius r . At time $t = 0$, the system is at rest in a Newtonian reference frame and the beam is undeformed. Subsequent to this initial time, the hub is made to rotate about a vertical axis $X - X$, passing through the center of the hub, in such a way that Ω , the angular speed of the hub, is given by

$$\Omega(t) = \begin{cases} (2/5) \left[t - (7.5/\pi) \sin(\pi t/7.5) \right] \text{ rad/sec} & 0 \leq t \leq 15 \text{ sec} \\ 6 \text{ rad/sec} & t > 15 \text{ sec} \end{cases}$$

which represents a smooth transition from zero hub motion to a constant angular speed of 6 rad/sec. The beam has a length L , Young's modulus E , shear modulus G , mass per unit length ρ , and a circular cross-section of area A and area moment of inertia I .

The solid line in the figure below shows the time history of the displacement of the beam tip, in the plane of rotation, relative to a line fixed in the hub and originally parallel to the centroidal axis of the beam. This result was obtained using the theory and algorithm presented in Refs.[1] and [2] with three assumed modes and the following parameter values

$$\begin{aligned} r &= 0 \text{ m} & \rho &= 1.2 \text{ kg/m} \\ L &= 10 \text{ m} & A &= 4 \times 10^{-4} \text{ m}^2 \\ E &= 7 \times 10^{10} \text{ N/m}^2 & G &= 3 \times 10^{10} \text{ N/m}^2 & I &= 2 \times 10^{-7} \text{ m}^4 \end{aligned}$$

All external forces were neglected and the assumed modal functions were chosen to be equal to the first three eigenfunctions of an identical uniform cantilever beam with its root fixed in a Newtonian reference frame. The numerical integration was carried out using a 4th - 5th order, variable step-size, Runge-Kutta-Merson method with a print step and initial time step of .03 seconds and an error tolerance of 1×10^{-6} . The dashed line result was produced with an algorithm based on the assumed-mode formulation utilized in most flexible multibody programs. This result was verified by Fidelis Eke [(818) 354-2916] at Jet Propulsion Labs using DISCOS.

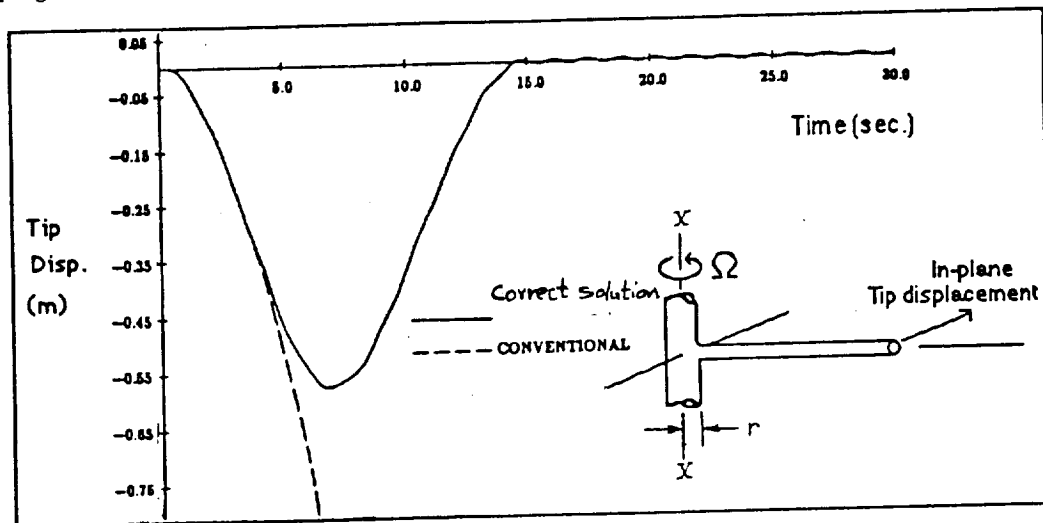
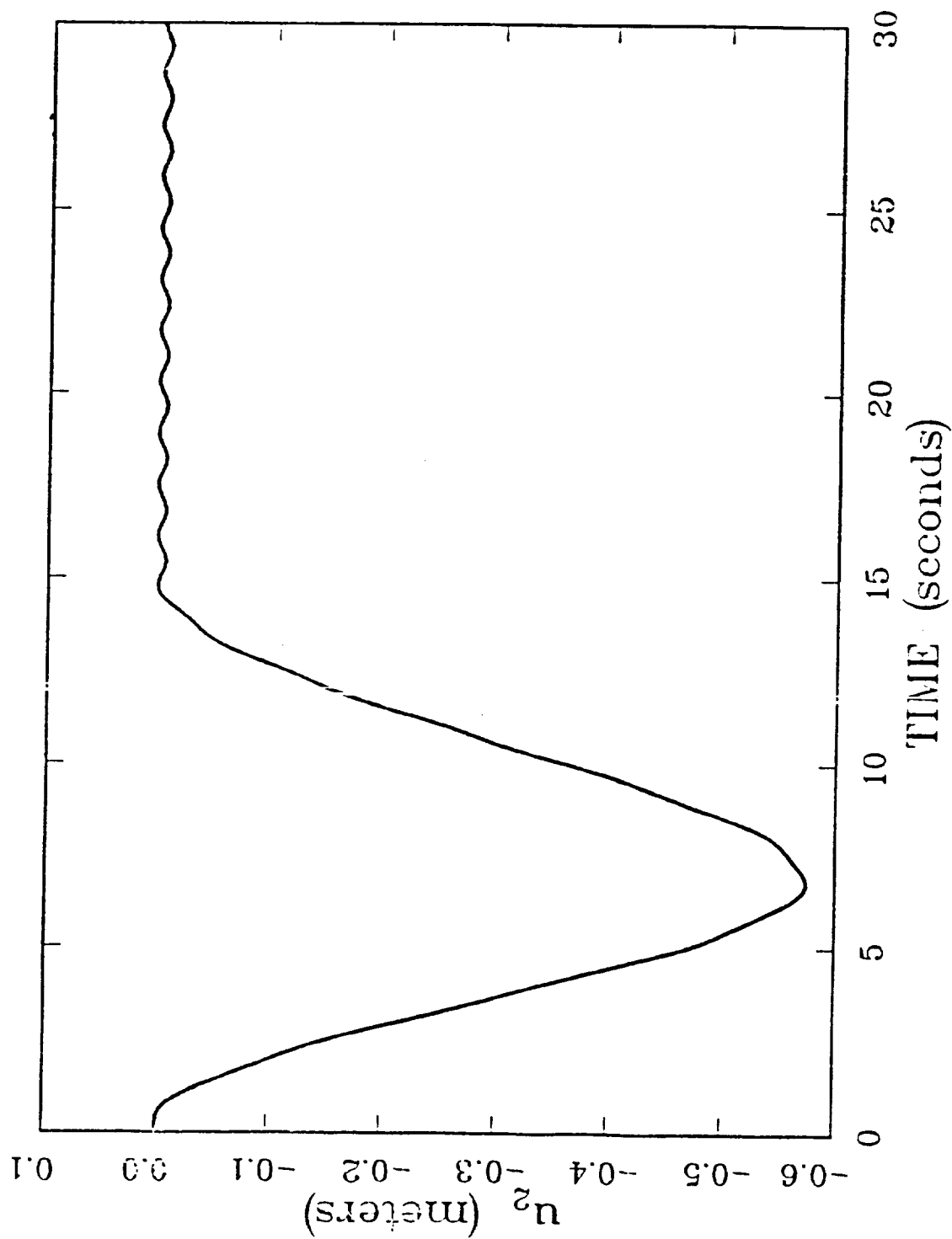


Fig. 1 Spin-up of Homogeneous Uniform Cantilever Beam

- [1] Ryan, R.R., "Flexibility Modeling Methods in Multibody Dynamics," Ph.D. dissertation, Stanford University, Available from University Microfilms, Ann Arbor, Michigan, 1986.
- [2] Kane, T.R., Ryan, R.R., and Banerjee, A.K., "Dynamics of a Cantilever Beam Attached to a Moving Base," *Journal of Guidance, Control, and Dynamics*, Vol.10, No.2, March-April, 1987.

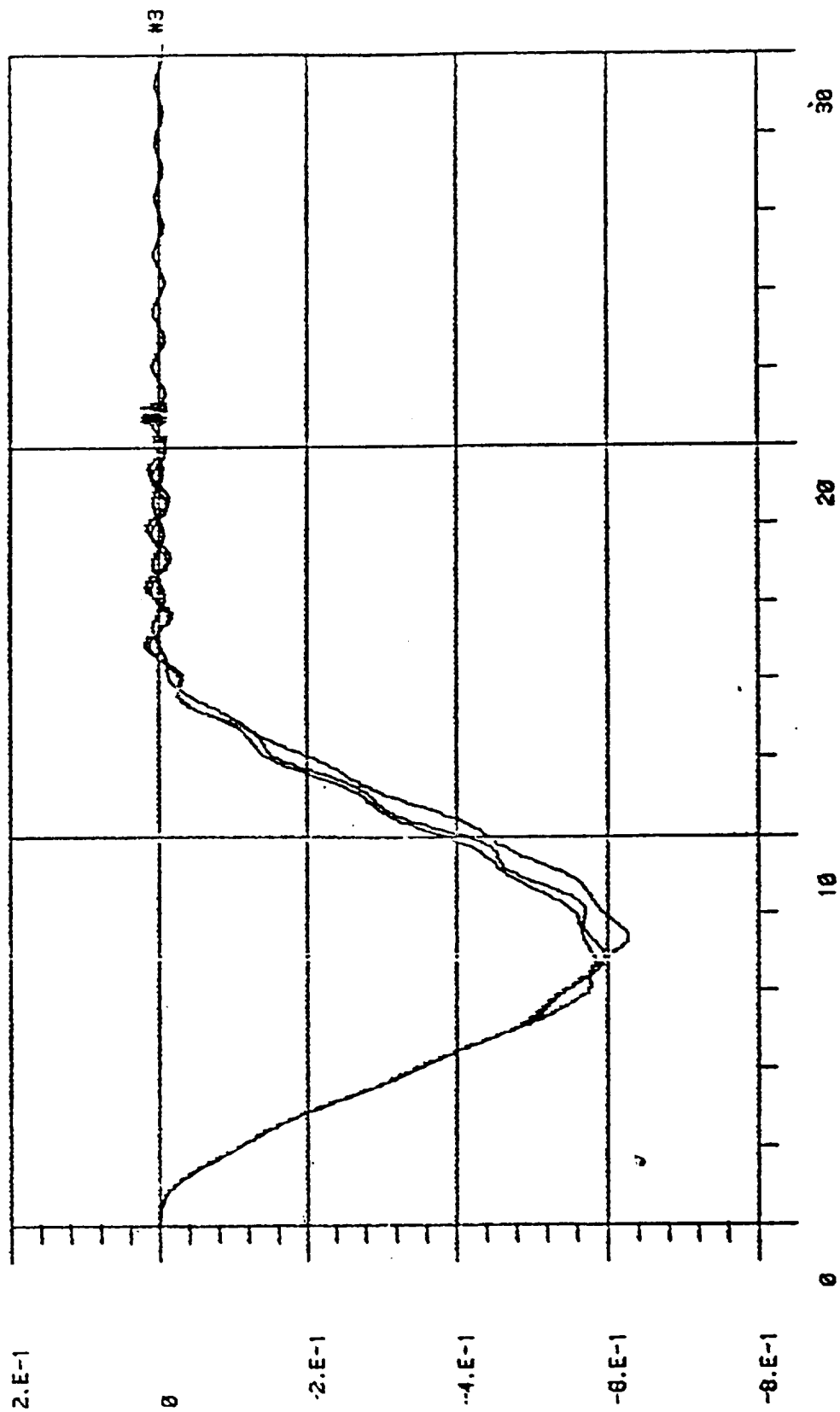
UNIFORM BEAM SPIN-UP PROBLEM

BEAM TIP DEFLECTION VS. TIME



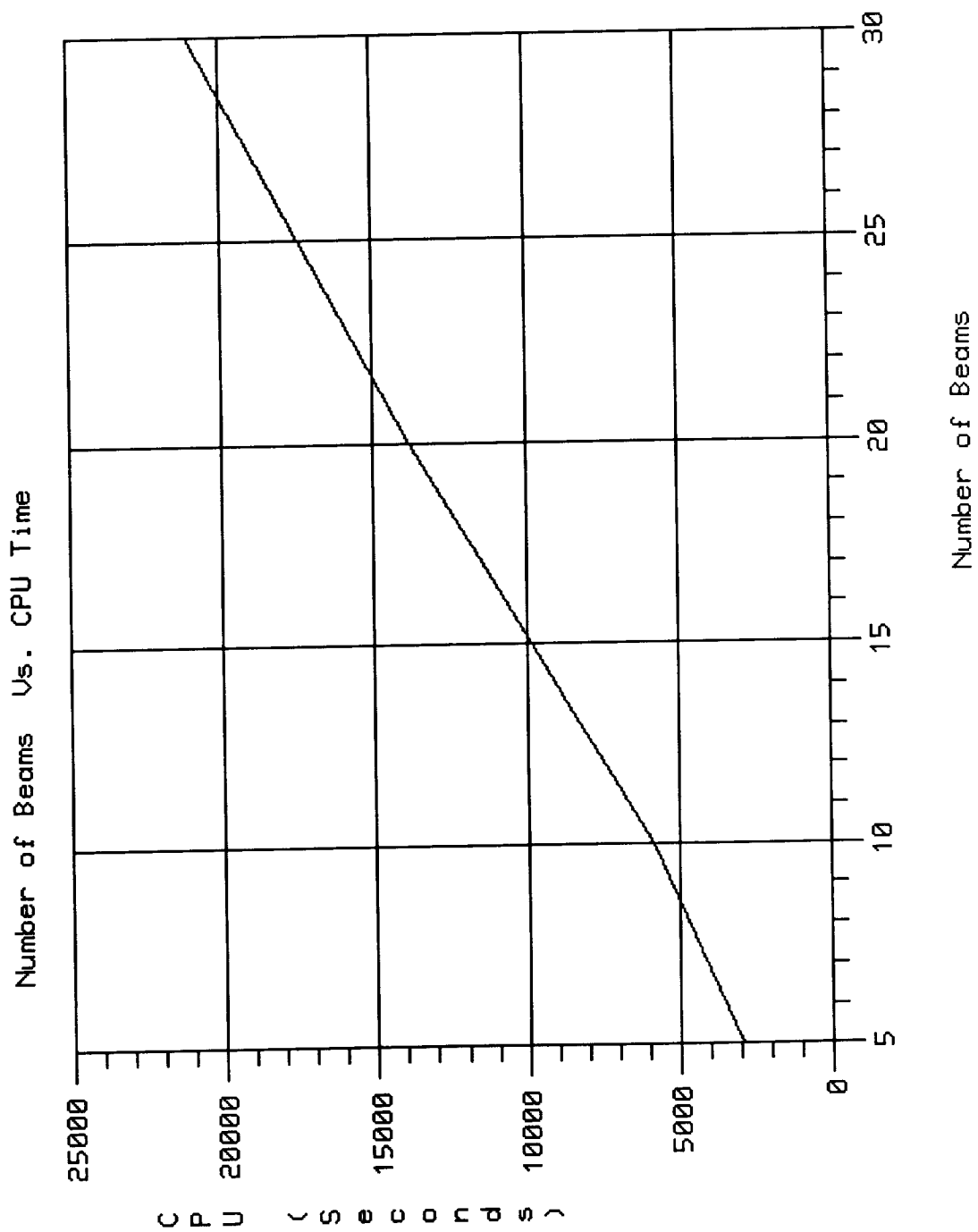
UNIFORM BEAM SPIN - UP PROBLEM : ADAMS APPROACH

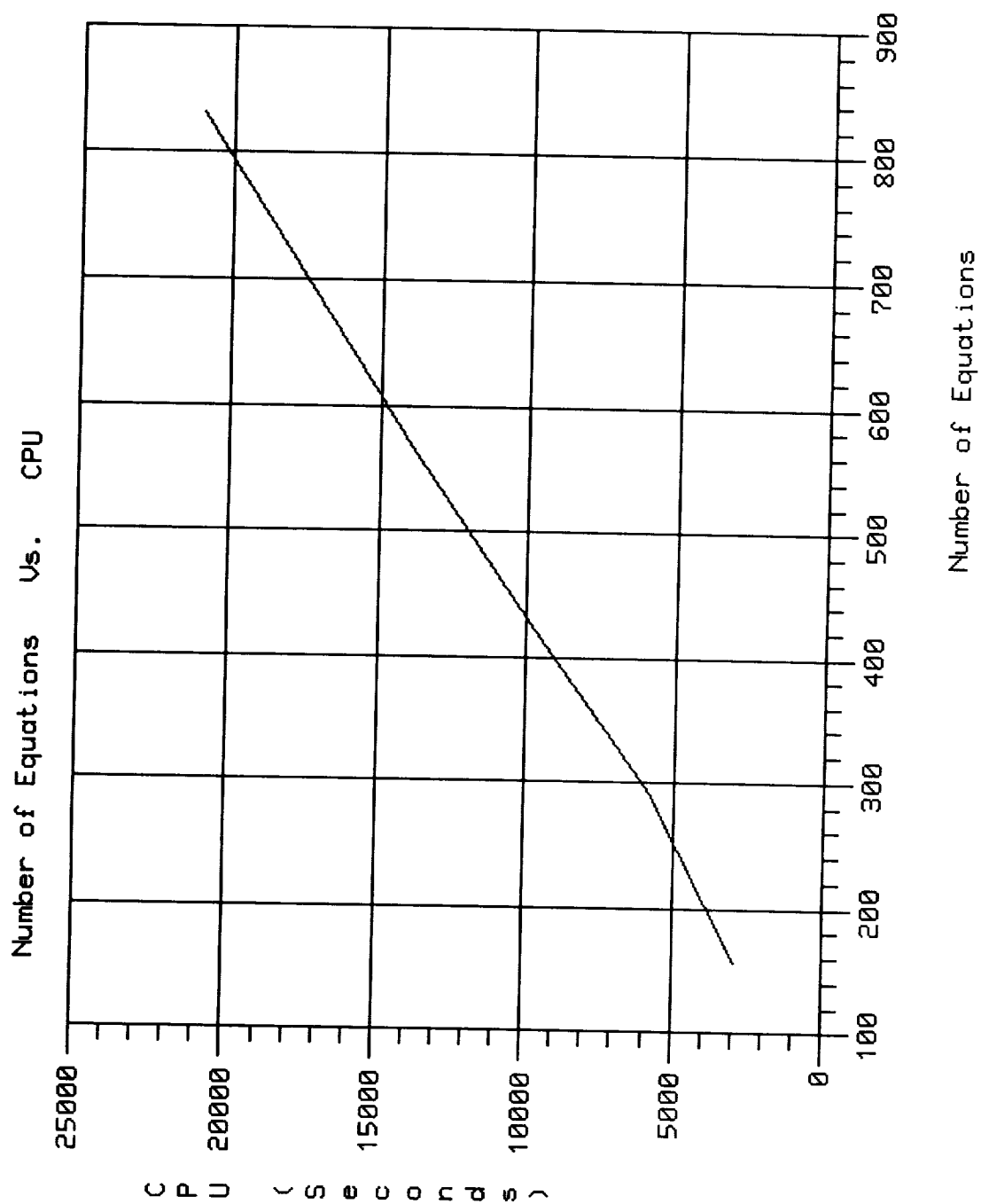
BEAM TIP DEFLECTION Vs. TIME



EFFICIENCY OF METHOD

- **THIS METHOD IS USABLE FOR SMALL TO MEDIUM SIZE PROBLEMS
(MEDIUM = 300 RIGID AND FLEXIBLE DOF)**
- **FOR LARGER PROBLEMS IT MAY PROVE TO BE MORE CPU INTENSIVE
THAN DESIRABLE.**
- **THE CPU TIME TAKEN FOR A SIMULATION IS LINEARLY
PROPORTIONAL TO THE NUMBER OF FLEXIBLE BEAMS IN
THE SYSTEM**

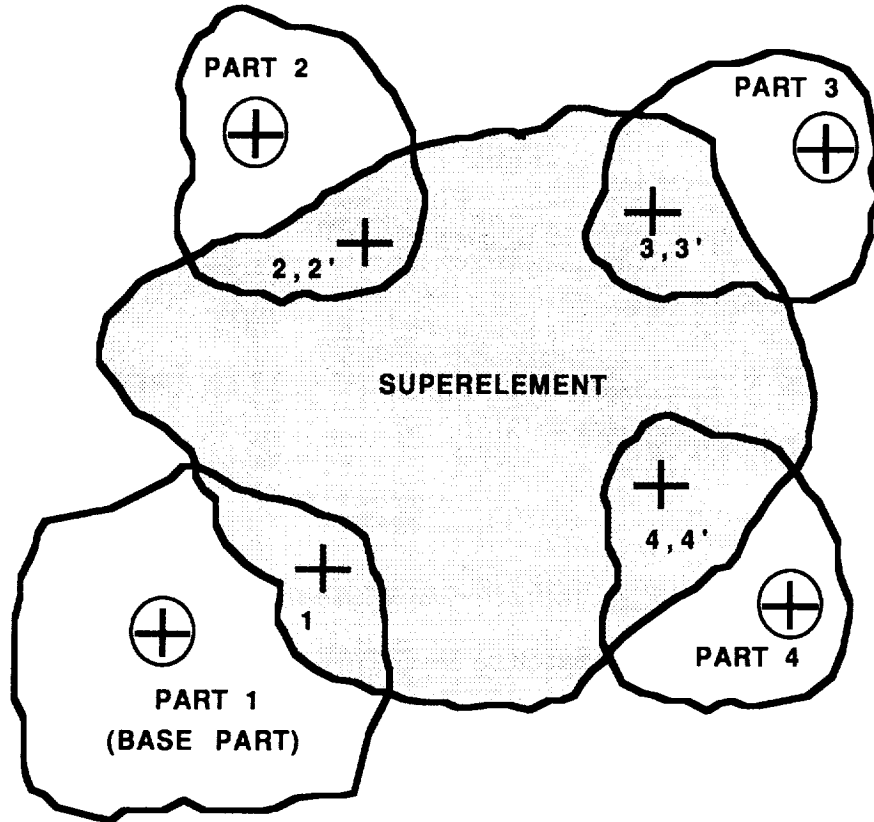




EASE OF USE

- **THE RESULTING PROGRAM IS EXTREMELY EASY TO USE.**
- **USERS DO NOT NEED STRONG FEA BACKGROUND TO CREATE MODELS OF STRUCTURES**
- **RECOGNITION AND SELECTION OF PROPER MODES IN AN ART. THE RESULTS ARE ONLY AS GOOD AS THE SELECTED MODES. DIFFICULTY ALLEVIATED IN THIS APPROACH.**

GENERALIZATION TO SUPERELEMENTS



NODES

- LOCATION OF NODES 2, 3, 4 WRT. TO A KNOWN REFERENCE FRAME
- MASS AND INERTIA PROPERTIES FOR EACH NODE OBTAINED FROM MASS MATRIX.

LOCATION AND ORIENTATION OF COORDINATE SYSTEMS

2 ON PART 2

2' ON PART 1

3 ON PART 3

3' ON PART 1

4 ON PART 4

4' ON PART 1

ASSEMBLY

- PRE-TENSION AND INITIAL DISPLACEMENTS
AT CONNECTION POINTS 2-2', 3-3', 4-4'

FLEXIBILITY PROPERTIES

- STIFFNESS MATRIX
- DAMPING MATRIX

DISPLACEMENT COORDINATES

$${}_1\Delta = [{}_1\Delta_{2'2} \quad {}_1\Delta_{3'3} \quad {}_1\Delta_{4'4}]^T$$

$$\mathcal{B} = [\mathcal{B}_{2'2} \quad \mathcal{B}_{3'3} \quad \mathcal{B}_{4'4}]^T$$

VELOCITY COORDINATES

$${}_1\mathcal{V} = [{}_1\mathcal{V}_{2'2} \quad {}_1\mathcal{V}_{3'3} \quad {}_1\mathcal{V}_{4'4}]^T$$

$${}_1\mathcal{Q} = [{}_1\mathcal{Q}_{2'2} \quad {}_1\mathcal{Q}_{3'3} \quad {}_1\mathcal{Q}_{4'4}]^T$$

FORCE DEFINITION AT COORDINATE SYSTEMS 2, 3, 4

$${}_1\mathbf{E} = [{}_1\mathbf{E}_2 \quad {}_1\mathbf{E}_3 \quad {}_1\mathbf{E}_4]^T$$

$${}_1\mathbf{T} = [{}_1\mathbf{T}_2 \quad {}_1\mathbf{T}_3 \quad {}_1\mathbf{T}_4]^T$$

FORCE COMPUTATION AT COORDINATE SYSTEMS 2, 3, 4

$$\begin{aligned} {}_1\mathbf{F} = & -[\mathbf{K}_{11} {}_1\Delta + \mathbf{K}_{12} \mathbf{B}] \\ & -[\mathbf{C}_{11} {}_1\mathbf{Y} + \mathbf{C}_{12} {}_1\mathbf{\omega}] + \mathbf{F}_0 \end{aligned}$$

$$\begin{aligned} {}_1\mathbf{T} = & - [\mathbf{K}_{21} {}_1\Delta + \mathbf{K}_{22} \mathbf{B}] \\ & - [\mathbf{C}_{21} {}_1\mathbf{Y} + \mathbf{C}_{22} {}_1\mathbf{\omega}] + \mathbf{T}_0 \end{aligned}$$

FORCE AT COORDINATE SYSTEM 1

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \vec{\mathbf{F}}_4 = \vec{0}$$

$${}_1\mathbf{F}_1 = - [{}_1\mathbf{F}_2 + {}_1\mathbf{F}_3 + {}_1\mathbf{F}_4]$$

$$\vec{\mathbf{T}}_1 + \vec{\mathbf{T}}_2 + \vec{\mathbf{T}}_3 + \vec{\mathbf{T}}_4 + \vec{\mathbf{S}}_{12} \times \vec{\mathbf{F}}_2 + \vec{\mathbf{s}}_{13} \times \vec{\mathbf{F}}_3 + \vec{\mathbf{s}}_{14} \times \vec{\mathbf{F}}_4 = \vec{0}$$

$${}_1\mathbf{T}_1 = - [{}_1\mathbf{F}_2 + {}_1\mathbf{F}_3 + {}_1\mathbf{F}_4]$$

$$- [{}_1\tilde{\mathbf{s}}_{12} {}_1\mathbf{F}_2 + {}_1\tilde{\mathbf{s}}_{13} {}_1\mathbf{F}_3 + {}_1\tilde{\mathbf{s}}_{14} {}_1\mathbf{F}_4]$$